2019

MATHEMATICS

(Major)

Paper : 2.1

(Coordinate Geometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) What is the locus represented by the equation $x^2 - 5xy + 6y^2 = 0$?

(b) What is the angle between the lines represented by the equation $x^2 - y^2 = 0$?

(c) Write down the parametric equations of a parabola.

(d) Write down the direction cosines of Y-axis.

(e) About which axis the parabola $x^2 = 4by$ is symmetric?

(f) What are the direction ratios of the normal to the plane $2x + y + z = 1$?

(g) What is the shortest distance between two coplanar lines?

(h) Write down the centre and radius of the sphere given by the equation

$$x^2 + y^2 + z^2 + 2x - 4y + 2z - 3 = 0$$

(i) Define conjugate planes.

(j) Define enveloping cylinder.

2. Answer the following questions : 2×5=10

(a) If the axes be turned through an angle $\tan^{-1} 2$, what does the equation $4xy - 3x^2 = a^2$ become?

(b) Find the value of k so that

$$kxy - 8x + 9y - 12 = 0$$

may represent pair of straight lines.

(c) Find the equation of the cone whose vertex is at the origin and guiding curve is given by $x = a$, $y^2 + z^2 = b^2$.

(d) Find the equation of the sphere through the circle

$$x^2 + y^2 + z^2 = 9$$
$$2x + 3y + 4z = 5$$

and the origin.

(e) Find the equation of the right circular cylinder whose axis is the line $\frac{x}{1} = \frac{y}{0} = \frac{z}{-2}$ and radius is $\sqrt{7}$.

3. Answer any *two* parts : 5×2=10

(a) Show that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel straight lines, if $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$.

(b) Choose the new origin (h, k) such that the equation $5x^2 - 2y^2 - 30x + 8y = 0$ may reduce to the form $Ax'^2 + By'^2 = 1$.

(c) Find the locus of the poles of chords of the parabola $y^2 = 4ax$ which subtends a right angle at the vertex.

(d) Prove that the middle points of the chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ parallel to the diameter $y = mx$ lie on the diameter $a^2my = b^2x$.

4. Answer any *two* parts : 5×2=10

(a) If by a transformation from one set of rectangular axes to another with the same origin the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then prove that $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$, where (x, y) and (x', y') are the coordinates of the same point referred to the two sets of axes.

(b) Prove that the straight line $y = mx + c$ touches the parabola $y^2 = 4a(x + a)$, if

$$c = ma + \frac{a}{m}$$

(c) Prove that the line $lx + my = n$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

(d) Find the equation of the tangent to the hyperbola $4x^2 - 9y^2 = 1$ which is parallel to the line $4y = 5x + 7$.

5. Answer any *four* parts : 5×4=20

(a) Find the equation to the hyperbola whose asymptotes are given by the equations $x+2y+3=0$ and $3x+4y+5=0$, and which passes through the point (1, – 1).

(b) A variable plane is at a constant distance p from the origin and meets the axes in A, B, C. Show that the locus of the centroid of the tetrahedron $OABC$ is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$.

(c) Obtain the shortest distance between the lines

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$

and $$\frac{x-\alpha'}{l'}=\frac{y-\beta'}{m'}=\frac{z-\gamma'}{n'}$$

(d) Prove that the equation of the plane containing the line $\frac{y}{b}+\frac{z}{c}=1$, $x=0$ and parallel to the line $\frac{x}{a}-\frac{z}{c}=1$, $y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$. If $2d$ is the shortest distance between the given lines, prove that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{d^2}$.

(e) Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 = 49$ which passes through the line

$$2x + z - 21 = 0 = 3y - z + 14$$

(f) Prove that the centres of spheres which touch the lines $y = mx$, $z = c$; $y = -mx$, $z = -c$ lie upon the surface

$$mxy + cz(1 + m^2) = 0$$

6. Answer any *four* parts : 5×4=20

(a) Find the equation of cone having the three coordinate axes as generators.

(b) Prove that from any point six normals can be drawn to the conicoid

$$ax^2 + by^2 + cz^2 = 1$$

(c) Find the equation to the polar planes of the conicoid $ax^2 + by^2 + cz^2 = 1$ with respect to the point (α, β, γ).

(d) Find the equations of the tangent planes to the conicoid $2x^2 + 3y^2 - 4z^2 = 1$ which are parallel to the plane $x - 3y + z = 0$.

(e) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and guiding curve is the curve

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 = z$$

(f) Find the equation of the cone whose vertex is (α, β, γ) and guiding curve is $y^2 = 4ax,\ z = 0.$

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