UNIT 6 PRODUCTION WITH ONE VARIABLE INPUT

Structure

6.0 OBJECTIVES

After going through this unit, you will be able to :

- state the concept of total product, average product and marginal product;
- explain the nature and relationship of total, average and marginal product curves;
- analyse the operation of the law of variable proportions; and
- identify the three stages of production.

6.1 INTRODUCTION

For the purpose of production, we require a combination of various inputs or factors of productions. It is only with the joint efforts of these inputs (like labour, machines, land, raw materials etc.) that output is produced. Normally, production is carried out under conditions of variable proportions which implies that the rate of input quantities may vary. Fixed proportions production means that there is only one ratio of inputs that can be used to produce a good. For example, only one driver can work one truck. In this case, the ratio of driver and truck is technologically determined and is fixed. It is beyond the capabilities of the producer to change it. However, the ratio of land and labour in agriculture can be changed and is thus regarded as variable. In the short run, not all inputs are variable. In the long run, however, all inputs are variable and the ratio of inputs may also vary. This is the case of technological Progress. In this unit, we shall focus only on short run production. In the short run, for the

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purpose of analysis, it is often assumed that only one input is variable and all other inputs are fixed. We shall follow this convention.

6.2 TOTAL, AVERAGE AND MARGINAL PRODUCTS

At the outset we shall explain the concept of total, average and marginal products. The short run production function, whether it is shown as a table, a graph or as a mathematical equation, gives the total output obtainable from different quantities of the variable inputs given a specified amount of the fixed input. Let us now consider the case in which capital is fixed, but labour is variable, so that the firm can produce more output by increasing the labour input. For example, consider a firm manufacturing garments. It has a fixed amount of equipment, but it can hire more or less labour to operate the machines. For decision making, the firm's manager (or owner) must know how the amount of total output or product (Q) increases (if at all) as the labour input (L) increases. Table 6.1 provides this information about the production function.

Table 6.1 shows the output that can be produced with different amounts of labour and with capital fixed at 5 units. The first column shows the fixed amount of capital, the second shows the amounts of labour from zero to 10 units and the third shows total product or output. From the table, it is clear that when labour input is zero, output is zero because capital alone cannot produce anything. Then, upto a labour input of seven units output increases first at an increasing rate and then at a decreasing rate in response to increased use of labour. The eighth unit of labour input does not raise output. Whether firm applies 7 or 8 units of labour input to a fixed amount of capital input, total output remains 224 units. Beyond this point using more units of labour input is counter productive because output declines as use of labour is increased.

Table 6.1: Production with One Variable Input

Amount of Capital (K)	Amount of Labour (L)	Total Product or Output (Q)	Average Product (Q/L)	Marginal Product ($\Delta Q/\Delta L$)
5	0	0	--	--
5	1	20	20	20
5	2	60	30	40
5	3	120	40	60
5	4	160	40	40
5	5	190	38	30
5	6	216	36	26
5	7	224	32	8
5	8	224	28	0
5	9	216	24	-8
5	10	200	20	-16

Although the figures provided in Table 6.1 are hypothetical, the general relationship they indicate is common. To examine the relationship further, we introduce the concepts of average product and marginal product of an input.

The average product (or average physical product) of an input can be defined as total output (or total product) divided by the amount of input used to produce that output. For example, 4 units of labour input produce 160 units of output, so the average product of labour is 40 units of output per worker at that level of employment. In a more general way, we may express

$$AP_L = \frac{Q}{L}$$

where, AP_L = average product of labour

Q = total output or total product

L = amount of labour

The fourth column in Table 6.1 shows the average product of labour (AP_L). The average product for each quantity of labour is derived by dividing total output shown in column 3 by corresponding amount of labour in column 2 that produces each output level. In our illustration, the average product of labour increases initially but when labour input exceeds 4 units, it tends to fall.

The marginal product (or marginal physical product) of an input is defined as the change in total output due to a unit change in the use of an input while quantities of other inputs are held constant. For example, with capital fixed at 5 units when the amount of labour increase from 3 to 4 units, total output rises from 120 to 160 units or by 40 units. So the marginal product of labour, when fourth unit of labour input is employed, is 40 units of output. We may thus generalise,

$$MP_L = \frac{\Delta Q}{\Delta L}$$

where, MP_L = Marginal product of labour

ΔQ = Change in output

ΔL = Change in labour input

In Table 6.1, the fifth column shows the marginal product of labour. It may be noted that like the average product, the marginal product increases initially and then falls and finally becomes negative. In the present example, the marginal product of labour becomes negative when labour input exceeds 8 units. This happens when the variable input is used too intensively with the fixed input.

The marginal product is greater than average product when average product is rising, equals average product when average product is at maximum, and is less than average product when average product is falling.

This proposition is, in fact, true of all marginal and average relationships.

6.3 TOTAL, AVERAGE AND MARGINAL PRODUCT CURVES

Fig. 6.1 plots the information provided in Table 6.1 (it has been assumed in drawing the graphs that both labour input and the product are divisible into smaller units and thus the relationships are smooth curves rather than discrete points). The total product curve shown in Fig. 6.1 indicates how the total

product varies with the quantity of labour input used. As indicated in Table 6.1, Fig. 6.1 a also shows that first the total output increases at an increasing rate upto point E as more labour is used. The point E where total product stops increasing at an increasing rate and begins increasing at a decreasing rate is called the **point of inflexion**. Total product reaches a maximum at 224 units when 7 units of labour input are used. The use of an additional unit of labour input at this stage does not lead to any increase in total product. Beyond this point, further use of labour input results in a fall in total product.

That portion of total product curve (TP) is shown by dashed segment which indicates a decline in output as a result of increased employment of labour. In Fig. 6.1 a when labour input is expanded beyond eighth unit, output falls which means that production is not technically efficient and is thus not a part of the production function.

Fig. 6.1 b shows the average and marginal product curves for labour. (The units of the vertical axis have been changed from output per period of time to output per unit of labour). Hence, average product and marginal product curves measure the output per unit of labour. It may be noted that as the use of labour input increases, initially the marginal product of labour increases, reaches a maximum at 3 units of labour, and then declines. The marginal product of labour in our example becomes zero at 8 units of labour and thereafter turns negative. However, technical efficiency rules out the possibility of negative marginal products and is, therefore, not a part of the production function. The average product of labour also increases initially, reaches a maximum at 4 units of labour input, and then declines.

Relationship between MP and AP Curves:

Let us now consider the relationship between the marginal and average product curves. As is true of all marginal and average curves, there are definite relationships between the marginal and average product curves.

i) When marginal product increases, average product also increases though at a rate lower than that of the marginal product. It is important to note in this context that even when marginal product starts declining but remains greater than the average product, the latter shows a tendency to increase.

ii) When the average product is maximum, the marginal product is equal to it. This is the reason why the marginal product curve intersects the average product curve at its highest point.

iii) Beyond this point, when the marginal product declines, it also pulls down the average product. However, the rate of decline in the average product is less than that of the marginal product.

Relationship between TP and MP Curves

The relationship between the total product curve and the marginal product curve can be stated as under:

i) As long as marginal product is positive, total product curve will continue to rise.

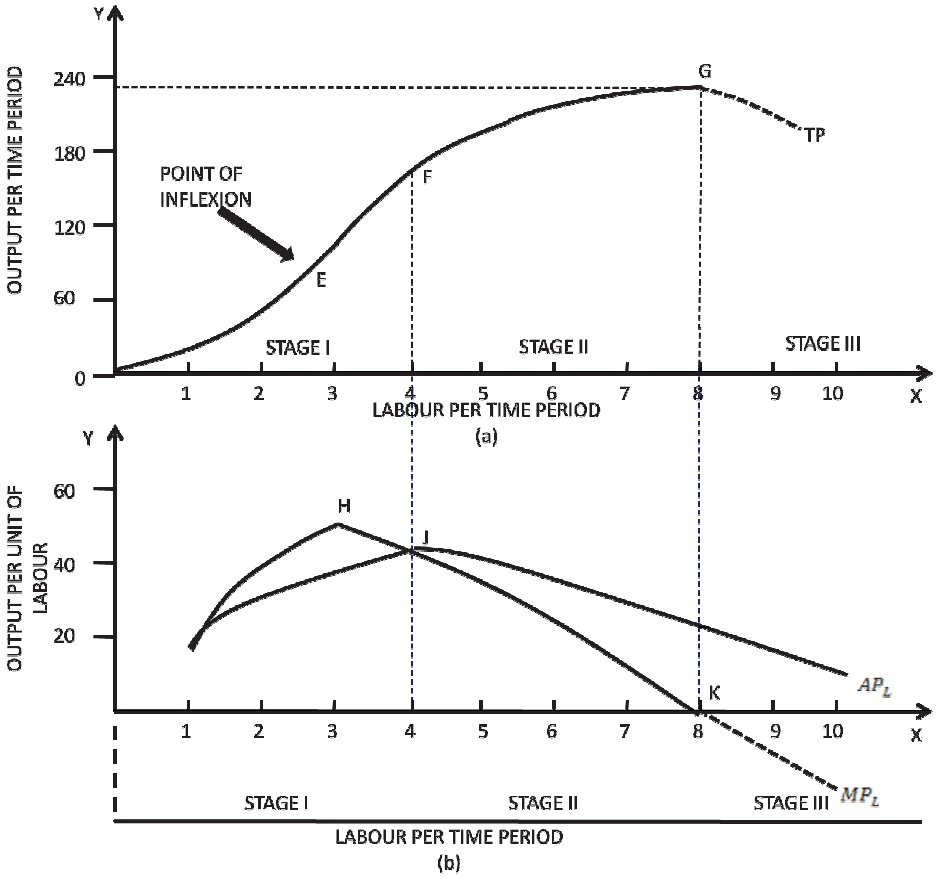

Fig 6.1: Production with one variable input (labour). In the upper part of the figure, the total product curve (TP) of labour is shown. The lower part of the figure shows how average product curve (AP) of labour and marginal product curve (MP) of labour are obtained with the help of information contained in the upper part

ii) When marginal product is zero, total product curve reaches its highest point. It may be noted that when eighth unit of labour input is employed, marginal product of labour becomes zero and total product is at the maximum.

iii) Thereafter, marginal product of labour is negative and total product curve has a downward slope which means that total product falls.

Check Your Progress 1

1) Indicate the following statement as true (T) or false (F):

   i) The marginal product is greater than average product when average product is falling.

   ii) As long as marginal product is rising, total product curve will continue to rise.

2) Discuss the relationship between the marginal and average product curves.

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6.4 THE LAW OF VARIABLE PROPORTIONS: RETURNS TO A FACTOR

Knowledge regarding the conditions of production reveals that as more and more of some input is employed, all other input quantities being held constant, normally marginal and average product (of the variable input) increase upto a point. Thereafter, marginal product starts declining and this pulls down the average product also. In the production process generally land, capital equipment and buildings remain fixed in the short run while quantities of labour and raw materials can be conveniently varied. However, we may consider a case where amount of capital is fixed and the quantity of labour is increased.

i) In this case, initially the marginal product of labour will increase as its amount is increased and the marginal product will also pull up average product with it. In this situation, total product increases at an increasing rate.

ii) If the variable input, say, labour is further increased, marginal product stops increasing after a point. Therefore, the rate of increase of total product also shows a tendency to fall.

iii) Ultimately marginal product turns negative and this causes a fall in total product itself.

Since in the short run, changes in technology are ruled out, the tendency of marginal product to decline after a point is inevitable. This statement of trends in marginal product in response to changes in the quantities of a variable factor applied to a given quantity of a fixed factor is called the **law of diminishing returns**. It is also called the law of variable proportions because it predicts the consequences of varying the proportions in which factors of production are used. we can sum up the law of variable proportions as follows:

> **"As equal increments of one input are added, the inputs of other productive services being held constant, beyond a certain point the resulting increments of product will decrease, i.e, the marginal product will diminish."**

The law of variable proportions can be easily followed with the help of Table 6.1 and Fig. 6.1 which has been drawn on the basis of illustration given in Table 6.1. In Table 6.1, it has been assumed that capital is a fixed factor and its quantity remains unchanged at 5 units. Labour is the variable factor and its quantity increases from 1 to 10. It can be seen from Table 6.1.

i) As the amount of labour employed increases, the total output also increases until the seventh unit of labour is employed. Initially the increase in output takes place at an increasing rate because marginal product rises. This tendency is observed upto the point E where marginal product reaches a maximum. At point E, which is the point of inflexion, the rate of increase in total product switches from increasing to decreasing because marginal product begins to diminish. However, average product continues to increase until it reaches a maximum at point F on total product curve (point J on average product curve).

ii) When the amount of labour is further expanded, total product continues to increase though at a diminishing rate. Both marginal product and

average product remain positive, but both continue to diminish. Eventually, total product reaches a maximum at point G and the marginal product becomes zero (note point K in Fig. 6.1 b). The average product, however, remains positive but continues to diminish.

iii) Any attempt to increase output beyond this point by employing more units of labour will not be fruitful. In fact, it will be counter-productive because marginal product is negative which implies that total product diminishes.

Product curves such as the one shown in Fig. 6.1 are general representations of production function with fixed and variable inputs. To illustrate particular instances, similar product curves could be drawn, though each different from others in some way. The stage of increasing marginal product may be long or brief or can be totally absent. Moreover, when marginal product diminishes, the rate at which it happens may be different in each case. Table 6.2 sums up the law of variable proportions.

Table 6.2: Properties of Product Curves

Total Product	Marginal Product	Average Product	Figure 6.1
Stage I			
first increases at increasing rate	Increases	Increases	to point E
then rate of increase changes from increasing to diminishing	reaches a maximum, and then starts diminishing	continues increasing	at points E and H
Stage II			
continues to increase at diminishing rate	continues diminishing	reaches a maximum where it equals MP and then starts diminishing	at points F and J
reaches a maximum and then starts diminishing	becomes zero	continues diminishing	at points G and K
Stage III			
diminishes	is negative	continues diminishing	to right of points J and K

6.4.1 The Three Stages of Production

Normally when the amount of a variable input is expanded, the marginal product first rises and then falls and the product curves have the shapes shown in Fig. 6.1. Conventionally, these product curves are partitioned into three regions, shown as Stages I, II and III in Fig. 6.1.

Stage I is characterised particularly by the rising average product. In our example, Stage I occurs when labour is employed from 1 to 4 units. In Stage 1, total product first increases at an increasing rate and thus marginal product rises. It reaches a maximum at labour input of 3 units. When fourth unit of labour input is employed, diminishing returns set in implying that total product increases at a diminishing rate and the marginal product falls.

In **Stage II**, total product increases at a diminishing rate and thus both marginal product and average product decline. Marginal product being below the average product, pulls the latter down. The right-hand boundary of Stage II is at maximum total product where marginal product reaches zero. In our example, Stage II ranges from 4 to 8 units of labour.

In **Stage III**, total product falls and marginal product is negative. In our example, stage III occurs when labour is employed in excess of 8 units.

Actual Stage of Operation

The rational producer will operate in Stage II. It is not difficult to follow why production will not be done in Stage III. In Stage III, less output is produced by using more of the variable input which means that production costs would be higher in Stage III than they were in Stage II. Obviously, any rational producer will always avoid such inefficiencies in the use of production inputs.

In Stage I, average product of the variable input is increasing. Therefore, if the amount of variable input is doubled, the output more than doubles and the unit cost of producing output decreases. If a firm is operating in a competitive market, it would avoid producing in this stage because by expanding output it reduces the unit costs while the price it receives remains same for each additional unit sold. This means that total profits increase if production is expanded beyond the region of rising average product.

To sum up we can say: Initially, the variable factor-labour is not able to use all the capacities of the fixed factor, hence MP and AP remain low. For instance, one worker may not be able to make full use of the potential of a one hectare plot of land. But two workers, together are is a better position to work on that field. Hence rise in MP as Labour increases from 1 to 2.

Thus, any rational producer will operate in the second stage only when the law of diminishing marginal return operates. This is why the law of variable proportions is also called the Law of Diminishing Marginal Returns to a factor.

6.4.2 Explanation of Increasing Returns

According to modern economists, when in the initial stage of production quantity of the variable factor is increased, the tendency of increasing returns in production operates. The classical economists had also observed this tendency and had termed it as the Law of Increasing Returns. However, they felt that this law operated only in manufacturing industries. As against this, the modern economists believe that this law can operate in any area of economic activity. Below we give the views of Marshall (representing the former position) and Joan Robinson (representing the latter position) in this regard.

Marshall opined that the tendency of increasing returns operates only in the manufacturing industries. He believed that when the quantity of labour and capital employed in the manufacturing industries is increased, the scale of

production expands and this leads to a better organisation of production. In Marshall's own words:

> **"An increase in labour and capital leads generally to improved organisation, which increases the efficiency of the work of labour and capital... Therefore, in those industries which are not engaged in raising raw produce, an increase in labour and capital generally gives a return increased more than in proportion."**

Joan Robinson's explanation of the tendency of increasing returns is more scientific. She states:

> **"When an increased amount of any factor of production is devoted to a certain use, it is often the case that improvements in organisation can be introduced which will make natural units of the factor (men, acres or money capital) more efficient, so that an increase in output does not require a proportionate increase in the physical amount of the factors."**

1) The tendency of increasing returns operates not only in manufacturing industries but in all productive activities. Limiting the application of this tendency to manufacturing industries alone is wrong.

2) The tendency of increasing returns comes into operation because the efficiency of the factors of production is improved.

Let us now examine in detail why the tendency of increasing returns operates.

1) **Optimum combination of factors of production:** According to Joan Robinson, full exploitation of some indivisible factors of production is not possible until increased quantities of some other factors of production are employed. Therefore, when the producer engages a small quantity of different factors of production, an optimum proportion among them is not established and the level of production remains low. When he increases the quantities of those factors of production, which were employed less (in relation to the requirements of optimum production), marginal product increases till the point is reached where the factors are combined in optimum proportion. Naturally, at this point, output level is the maximum.

2) **Large size of fixed factors:** When the size of the fixed factors used for producing a given good is very large while the quantity of the variable factor used is very small, the level of efficiency remains very low. As more and more quantities of the variable factors are employed, marginal productivity increases (since the level of efficiency increases). For example, if only one person is working on a ten hectare plot of land, his productivity will be very low. As the number of workers increases, division of labour and specialisation will lead to increasing returns as marginal product will rise rapidly.

6.4.3 Explanation of Constant Returns

If even on continuously increasing the quantity of variable factors of production in a firm, the marginal product neither increases nor decreases but

remains constant, the tendency of constant returns is in operation. In fact, there is no industry in which increase in the quantity of variable factors of production yields constant returns permanently. According to Marshall, "if the actions of the law of increasing and diminishing returns are balanced, we have the law of constant returns."

Marshall feels that the operation of the law of constant returns is very limited. According to him, this law can operate only when there is a balance between the tendencies of increasing returns and diminishing returns. However, modern economists regard the area of operation of constant returns as fairly large. According to them, tendency of constant returns is generally found to operate before the tendency of diminishing returns sets in. In no field of productive activity increasing returns are obtained forever. Whether it is agriculture, manufacturing, industry or any other productive activity, the tendency of increasing returns can operate only up to a certain limit. After this limit is reached, constant returns operate for some time. From the point of view of the producer, this is an important stage because it exhibits an optimum combination of the factors of production. In this stage, marginal cost is the minimum. This is due to two reasons. First, the stage of constant returns is reached only when the tendency of increasing returns comes to an end so that there is no possibility of a further decline in marginal cost. Second, after the stage of constant returns, the stage of diminishing returns sets in. Therefore, the stage of constant returns is very significant from the point of view of the producers.

6.4.4 Explanation of Diminishing Returns

The diminishing returns stage is the most important of the three stages of the law of variable proportions. In Economics, the explanation of the law of diminishing returns is presented in two ways. The classical economists believed that this law applies only to agriculture. Basically accepting this position of the classical economists, the neo-classical economist Marshall had stated, "We say broadly that while the part which nature plays in production shows a tendency of diminishing returns, that part which man plays shows a tendency of increasing returns."

Modern economists like Joan Robinson, Stigler, etc. constitute the second category of economists. These economists regard the law of diminishing returns of far greater applicability than the classical economists. According to them, this law operates in all areas of productive activity.

Marshall had argued that this law operated only in agriculture. Therefore, he discussed it only in reference to agriculture. According to him,

> **"An increase in the capital and labour applied in the cultivation of land causes in general a less than proportionate increase in the amount of produce raised unless it happens to coincide with an improvement in the arts of agriculture."**

The implication is that when land is kept fixed in agriculture while the quantity of labour and capital applied on that land is increased, total production increases but not in the same proportion as the factors of production are increased. It increases by a lesser proportion. For example, if an agriculturist doubles the amount of labour and capital employed on a fixed plot of land, the total production will undoubtedly increase but it will not double itself. Due to

this reason agriculturists do not consider it profitable to continuously increase the application of other factors of production on their fixed plots of land. They know from their experience that unless there is some improvement in agricultural techniques, increased application of labour and capital on a fixed quantity of land leads to a situation of continuously declining marginal product.

Marshall has accepted two limitations of the law of diminishing returns as applied to agriculture:

1) **The law generally operates in agriculture:** Marshall was aware of the fact that the law of diminishing returns does not always operate in agriculture (hence the qualification that it generally operates in agriculture). In some cases when the agriculturist applies the first unit of labour and capital on his fixed plot of land, the fertility of the soil is not properly exploited. Accordingly, the level of production remains low. When the second unit of labour and capital is applied, output increases in a greater proportion. However, this tendency does not remain for long because the agriculturist soon finds that additional units of labour and capital start yielding a lower and lower marginal product. On account of the above reasons, Marshall was careful in pointing out that the law of diminishing returns operates generally in agriculture. However, in certain exceptional cases, it may not operate.

2) **There should be no improvement in agricultural techniques:** The law of diminishing returns operates only if there is no improvement in agricultural techniques. It is a law of static agriculture. If the agriculturist is able to expand irrigation facilities on his land, or make use of better seeds, better agricultural implements, more fertilisers, etc. or use new scientific methods in production, he can stall the operation of this law. Generally, an improvement in agricultural techniques leads to a more than proportionate increase in output corresponding to an increase in labour and capital.

As against the view of Marshall, modern economists like Joan Robinson, Stigler and Boulding regard the law of diminishing returns as more pervasive and universal. According to these economists, this law operates in all branches of productive activity. Accordingly, they have presented this law in a general fashion as would be clear from the definition of this law presented by Joan Robinson:

> **"The Law of Diminishing Returns, as it is usually formulated, states that, with fixed amount of any one factor of production, successive increases in the amount of other factors will after a point yield a diminishing increment of the product."**

From the above definition of the law by Joan Robinson, it is clear that she regards this law as of universal value and does not restrict its application to agriculture alone. According to her, this law operates in all branches of productive activity and the principal reason behind the operation of this law is that the optimum proportion between different factors of production breaks down sooner or later.

The law of diminishing returns is a logical necessity. When in any productive activity, the quantity of the variable factors of production employed with given

quantity of fixed factor of production is increased, the law of diminishing returns sets in after the point of optimum proportion has been reached. Initially, application of variable factors was sub-optional, given the size of fixed factor. Later, the expansion in use of variable factors leads to sub-optimality of a different kind: each doze or unit of variable factors have sub-optional quantity of fixed factor to work on.

Another important reason for the operation of the law of diminishing returns is that one factor of production (out of the various factors of production) is used in a fixed quantity. Had all the factors of production been available in abundance and had it been possible to increase their use in production to all conceivable limits, the law of diminishing returns would not operate. However, all factors of production land, labour, capital, enterprise, organisation, etc. are scarce and often the supply of one of these is taken to be fixed. It is this factor that results in diminishing returns.

Check Your Progress 2

1) Indicate the following statement as true (T) or false (F):

   i) In statge II of production, both marginal product and average product decline.

   ii) In stage III of production, marginal product is negative.

   iii) The law of diminishing returns operates only in agriculture.

2) State the law of diminishing marginal returns. There is a provision to the law that other things be held constant. What are these things?

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3) Explain the three stages of production. Why should a rational producer under competitive conditions produce in stage II?

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4) Explain the (i) law of increasing returns, (ii) law of constant returns.

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6.5 LET US SUM UP

In this unit we have focused on short run production assuming that only one input is variable and all other inputs are fixed. We then define total product, average product of an input and the marginal product of an input. We note that

total product in the case of production with one variable input first increases at an increasing rate as the amount of variable input expands and then switches to increasing with decreasing rate. Having reached a maximum, it eventually declines. We then explain the law of variable proportions. Conventionally the product curves drawn to depict the law of variable proportion are partitioned into three stages. In stage I, average product increase throughout, in stage II marginal product from the point where it equals average product falls throughout but remains positive; and in stage III total product fall and marginal product is negative. The diminishing returns stage is the most important of the three stages of the law of variable proportions.

6.6 REFERENCES

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4) John P Gould and Edward P Lazar, *Microeconomic Theory* (All India Traveller Bookseller, Sixth edition, 1996), Chapter 6.

6.7 ANSWERS OR HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) (i) (F); (ii) (T)

2) See Section 6.3

Check Your Progress 2

1) (i) F; (ii) (T); (iii) (F)

2) See Sub-section 6.4.4

3) See Sub-section 6.4.1

4) See Sub-section 6.4.2. for law of increasing returns and Sub-section 6.4.3 for law of constant returns.